Practical guide to loop integration
Exercise Sheet 2

AEC
ALBERT EINSTEIN CENTER
FOR FUNDAMENTAL PHYSICS

Summer Term 2024
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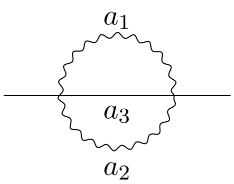
Issued: 23 April 2024

Exercise 1: Sunset diagram Consider the following integral

$$I(a_1, a_2, a_3; \mathcal{N}) = \mathcal{N} \times$$



$$= \int [\mathrm{d}k_1][\mathrm{d}k_2] \frac{\mathcal{N}}{\left[k_1^2\right]^{a_1} \left[k_2^2\right]^{a_2} \left[(k_1 - k_2 - p)^2 - m^2\right]^{a_3}} \tag{1}$$

with $p^2 = m^2 \neq 0$ and an arbitrary numerator $\mathcal{N}$.

a) Find a complete family. We know that it will need $\ell(1 + \ell + 2\rho)/2 = 5$ propagators.

b) Identify the sectors in which all integrals vanish.

c) Consider the IBP generated through $\partial_{k_1^\mu}(k_1^\mu I)$. Use it to show that

$$\int [\mathrm{d}k_1][\mathrm{d}k_2] \frac{k_2 \cdot p}{\left[k_1^2\right] \left[k_2^2\right] \left[(k_1 - k_2 - p)^2 - m^2\right]^2} = \frac{3-d}{2} \int [\mathrm{d}k_1][\mathrm{d}k_2] \frac{1}{\left[k_1^2\right] \left[k_2^2\right] \left[(k_1 - k_2 - p)^2 - m^2\right]} \tag{2}$$

d) Now find all six seed identities as a function of $a_1, \ldots, a_5$.

e) Implement Laporta's algorithm to solve the system up to $r = 4$ and $s = 1$ for sector 7 and its subsectors.

Exercise 2: Sunset diagram using computer codes

Consider again the same integral (1) but now with $p^2 = s \neq m^2$. Perform the reduction using `reduze` or `kira` for all integrals with $r \leq 4$ and $s \leq 1$.

a) Make a list of all 75 integrals you want to calculate using a computer program.

b) Perform the reduction of all integrals without specifying a basis of master integrals. How many master integrals do you find?

c) Consider the following possible choices of master integrals. Which ones do you prefer and why?

$$\vec{I}_1 = \begin{pmatrix} I(1,1,1;1) \\ I(1,1,2;1) \end{pmatrix}, \qquad \vec{I}_2 = \begin{pmatrix} I(1,1,1;1) \\ I(1,1,1;p\cdot k_1) \end{pmatrix}, \qquad \vec{I}_3 = \begin{pmatrix} I(1,2,1;1) \\ I(1,1,2;1) \end{pmatrix}. \tag{3}$$