Durham University

# Practical guide to analytic loop integration
# Exercise Sheet 4

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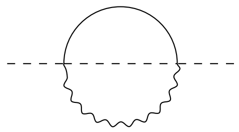
You might find the Mathematica packages `MB Tools` useful: `https://mbtools.hepforge.org/`. Consider using especially `MB.m`, `MBresolve.m`, and `barnesroutines.m`.

**Exercise 1:** Mellin Barnes expansion

In this example, we will consider once again a sunset diagram



$$= \int [\mathrm{d}k_1][\mathrm{d}k_2] \frac{1}{\left[k_1^2 - m^2\right]\left[k_2^2 - M^2\right]\left[(k_1-k_2-p)^2\right]} \tag{1}$$

with $p^2 = m^2$.

a) Use a single MB split to solve the Feynman integrals. There is no need to sum the MB series yet.

The MB will be of the form

$$I = \int_{-\mathrm{i}\infty}^{+\mathrm{i}\infty} \mathrm{d}z \left(\frac{m^2}{M^2}\right)^{-z} f(z) = \sum_{n=0}^{\infty} \left(\frac{m^2}{M^2}\right)^n f'(n)$$

with some $f(z)$ and $f'(n)$ assuming we have closed the contour on the correct side.

We can at this point decide that $0 < m \ll M$ and expand the integral. Conceptually, this is done by noting that the terms in the series of residues are suppressed by $(m^2/M^2)^n$. To expand to any order in $m^2/M^2$ we just truncate the series.

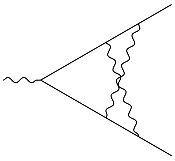
b) Calculate the integral up to $\mathcal{O}(m^4)$.

c) Solve the integral exactly in $m$ by calculating the full series, expand in $\epsilon$ using `HypExp`. Finally expand in $m$ to verify your result.

   *(for the adventurous)*

**Exercise 2:** Multiple Mellin Barnes

Consider the following non-planar integral



$$= \int [\mathrm{d}k_1][\mathrm{d}k_2] \frac{1}{\left[k_1^2\right]\left[k_2^2\right]\left[(k_1-p-q)^2\right]\left[(k_1-k_2)^2\right]\left[(k_1-k_2-q)^2\right]\left[(k_2-p)^2\right]}$$

with $p^2 = q^2 = 0$ and $(p+q)^2 = s$.

a) Solve the Feynman integration. This can be done using two Mellin Barnes splits

$$\frac{1}{(A_1+A_2+A_3)^\lambda} = \frac{1}{\Gamma(\lambda)} \int_{-\mathrm{i}\infty}^{+\mathrm{i}\infty} \mathrm{d}z_1 \mathrm{d}z_2 A_1^{z_1}\Gamma(-z_1) A_2^{z_2}\Gamma(-z_2) A_3^{-\lambda-z_1-z_2}\Gamma(\lambda+z_1+z_2)\,.$$

*Hint*: You might find the substitution $x_2 \to x_6 x_2$ useful.

b) Resolve the singularities and expand in $\epsilon$ up to $\epsilon^0$.

c) Use the Barnes Lemmas and PSLQ to solve the resulting integral