Practical guide to analytic loop integration
Exercise Sheet 2

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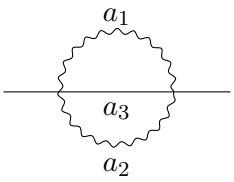
Exercise 1: Sunset diagram Consider the following integral

$$I(a_1,a_2,a_3;\mathcal{N}) = \mathcal{N} \times$$ 

$$= \int [\mathrm{d}k_1][\mathrm{d}k_2] \frac{\mathcal{N}}{\left[k_1^2\right]^{a_1}\left[k_2^2\right]^{a_2}\left[(k_1-k_2-p)^2-m^2\right]^{a_3}} \tag{1}$$

with $p^2 = m^2 \neq 0$ and an arbitrary numerator $\mathcal{N}$.

a) Find a complete family. We know that it will need $\ell(1+\ell+2\rho)/2 = 5$ propagators.

b) Identify the sectors in which all integrals vanish.

c) Consider the IBP generated through $\partial_{k_1^\mu}(k_1^\mu I)$. Use it to show that

$$\int [\mathrm{d}k_1][\mathrm{d}k_2] \frac{k_2\cdot p}{\left[k_1^2\right]\left[k_2^2\right]\left[(k_1-k_2-p)^2-m^2\right]^2} = \frac{3-d}{2}\int [\mathrm{d}k_1][\mathrm{d}k_2] \frac{1}{\left[k_1^2\right]\left[k_2^2\right]\left[(k_1-k_2-p)^2-m^2\right]} \tag{2}$$

d) Now find all six seed identities as a function of $a_1,\ldots,a_5$.

e) Implement Laporta's algorithm to solve the system up to $r=3$ and $s=1$ for sector 7 and its subsectors.