

48. *Herbstschule für Hochenergiephysik 2016*

Muon decay

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Motivation

Polarisation and γ^5

The radiative decay

The rare decay

Outlook

Motivation

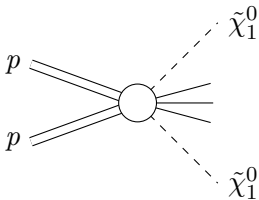
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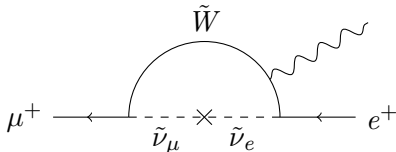
Outlook

New physics: lightning introduction



Direct search

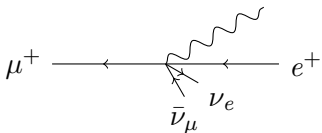
Sensitive up to $\mathcal{O}(10^3 \text{ GeV})$



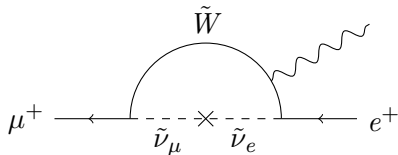
Indirect search

Sensitive up to $\mathcal{O}(10^{13} \text{ GeV})$ ¹

¹Model dependent



Radiative decay
 $(\mu \rightarrow \nu \bar{\nu} e + \gamma)$
 $E_{\nu \bar{\nu}} < \Delta E_{\text{exp.}}$



LFV decay
 $(\mu \rightarrow e \gamma)$

	Experimental	Theoretical (4-Fermi)
Normal $\mu \rightarrow \nu \bar{\nu} e$	TWIST ¹ $\mathcal{O}(10^{-4})$	NLO (polarised, MC) [Arbuzov 2001] NNLO (unpolarised, analytic) [Anastasiou, Melnikov, and Petriello 2007]
Radiative $\mu \rightarrow \nu \bar{\nu} e + \gamma$	MEG $\mathcal{O}(1\%)$	NLO (polarised, MC) [Fael, Mercolli, and Passera 2015]
Rare $\mu \rightarrow \nu \bar{\nu} e + e^+ e^-$	Mu3e ² $\mathcal{O}(10\%)$	LO (polarised, MC)

¹Michel parameters

²Proposed

Use muon as toy process: clean QED

- γ^5 : Treatment in dim-reg?
- Regularisation scheme dependency
- Large logs, esp. $\tau \rightarrow \nu \bar{\nu} e + \gamma$ (3.5σ deviation!) [[Fael, Mercolli, and Passera 2015](#)]
- No NLO for rare decay

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- Two equivalent ways of introducing polarisations dependence

- “Closing the trace” $u(p)\bar{u}(p) = (\not{p} + m) \frac{1 + \gamma^5 \not{\epsilon}}{2}$
- Massive spinor helicity formalism

$$u_{\pm}(p) = |\ell^{\pm}\rangle + \frac{m}{\langle \ell^{\pm} | n^{\mp} \rangle} |n^{\mp}\rangle$$

$$|k^{\pm}\rangle = \frac{1 \pm \gamma^5}{2} u(k)$$

n is related to s [\[See Ellis' lecture\]](#)

- Both introduce a γ^5 !

- $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ is not well defined in d dimensions
- There are at two sources of γ^5 :
 - The 4-Fermi vertex $j_{V-A}^\mu(a, b) = \bar{\psi}_a\gamma^\mu(1 - \gamma^5)\psi_b$: [Berman and Sirlin 1962]

$$j_{V-A}^\mu(a, b) = \underbrace{\bar{\psi}_a\gamma^\mu\psi_b}_{j^\mu} - \bar{\psi}_a\gamma^\mu\psi'_b$$

- $\psi'_b = \gamma^5\psi_b$ corresponds to an electron with $m = -m_e$.
- Polarisation: Spinor helicity formalism in FDH (external particles in $d = 4$)

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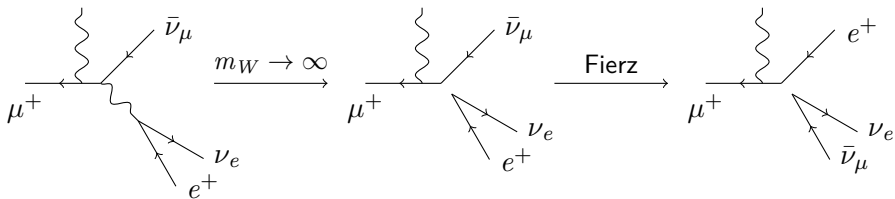
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Outlook

Fully differential NLO predictions for MEG @ PSI

- 4-Fermi interaction, fierzied at the Lagrangian

$$\mathcal{L} = \mathcal{L}_{\text{QED}} + \frac{G_F}{\sqrt{2}} j_{V-A}(\mu, e) \cdot j_{V-A}(\nu_\mu, \nu_e)$$

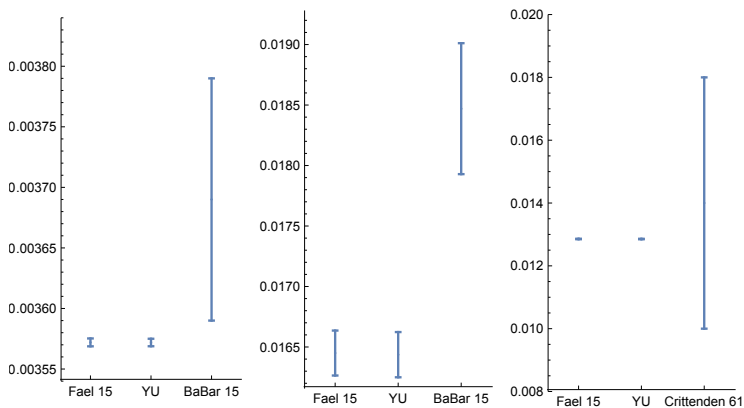


- Get amplitudes from GoSam [[Cullen et al. 2014](#)]
- FKS subtraction [[Frixione, Kunszt, and Signer 1996](#)]
- Custom phase spaces for increased stability and FKS
- (Almost) original VEGAS for integration [[Lepage 1980](#)]

Branching ratio: Experimental comparison

$$\delta\text{BR}^{\text{NNLO}} \approx \frac{\alpha}{\pi} \log \frac{m}{M} \log \frac{\omega_0}{M} \text{BR}^{\text{NLO}}$$

[Fael, Mercolli, and Passera 2015]



$$\tau \rightarrow \nu\bar{\nu}\mu + \gamma$$

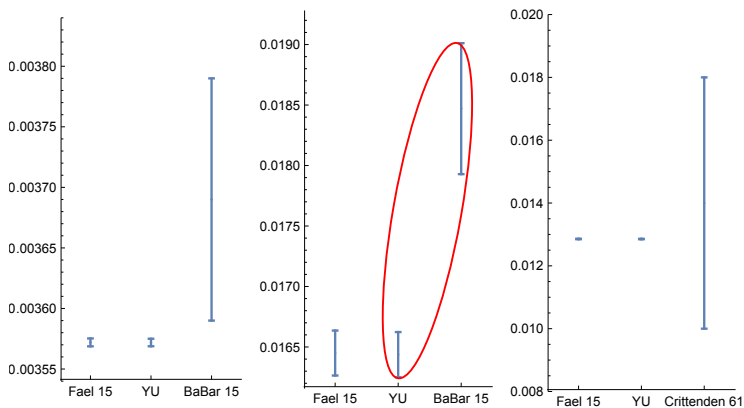
$$\tau \rightarrow \nu\bar{\nu}e + \gamma$$

$$\mu \rightarrow \nu\bar{\nu}e + \gamma$$

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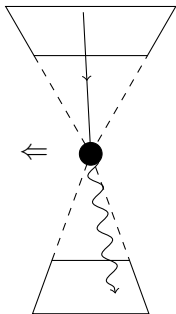


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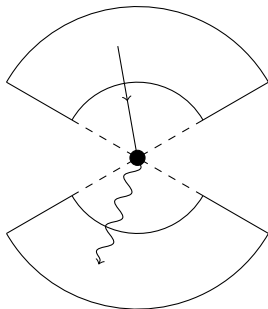
$\mu \rightarrow \nu \bar{\nu} e + \gamma$

Theorist's version of the MEG detector @ PSI



$$E_{\gamma} > 40 \text{ MeV}$$

$$|\cos \theta_e| < 0.5$$



$$E_e > 45 \text{ MeV}$$

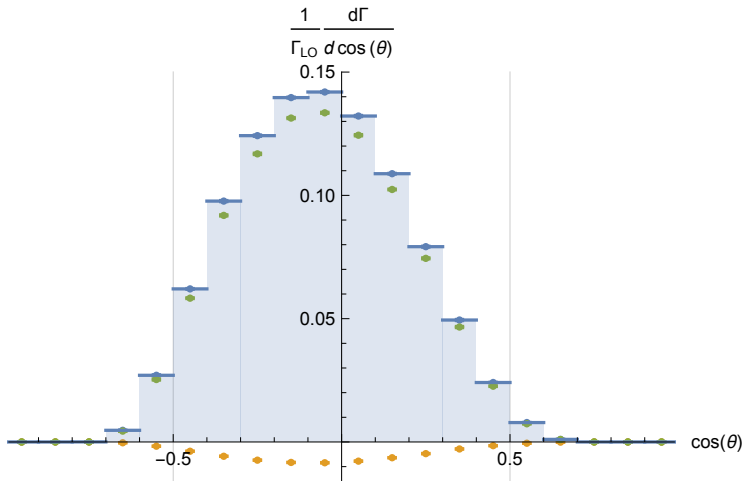


Fig.: Angular distribution:

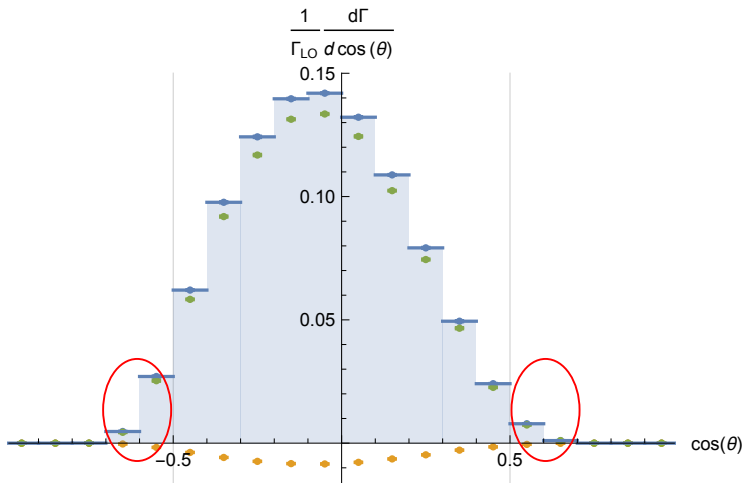


Fig.: Angular distribution: MEG cuts on the electron loose 4.10% of the events

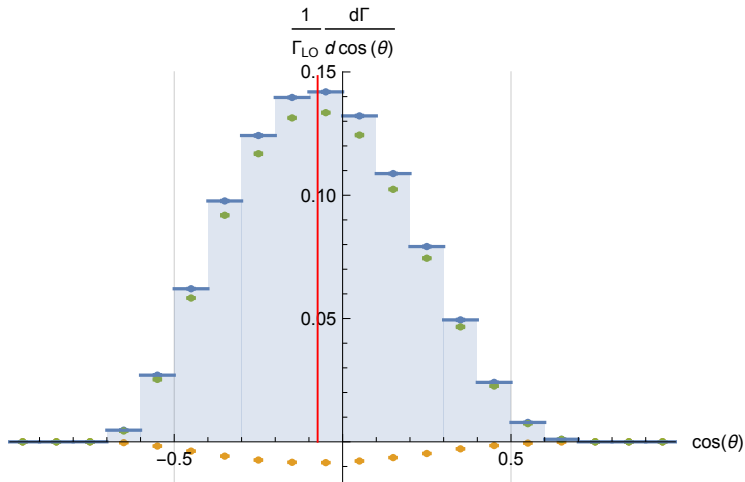


Fig.: Angular distribution: Polarised source $\langle \cos \theta_c \rangle \approx -0.063 < 0$
 corresponding to $\langle \theta_c \rangle \approx 93.62^\circ$

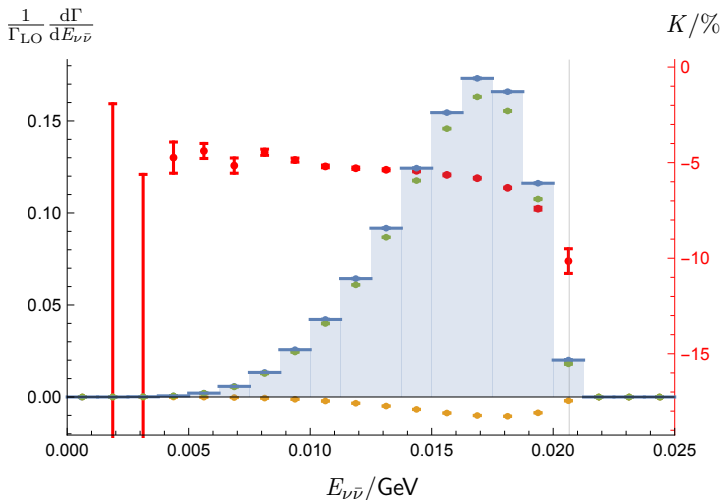


Fig.: Neutrino spectrum:

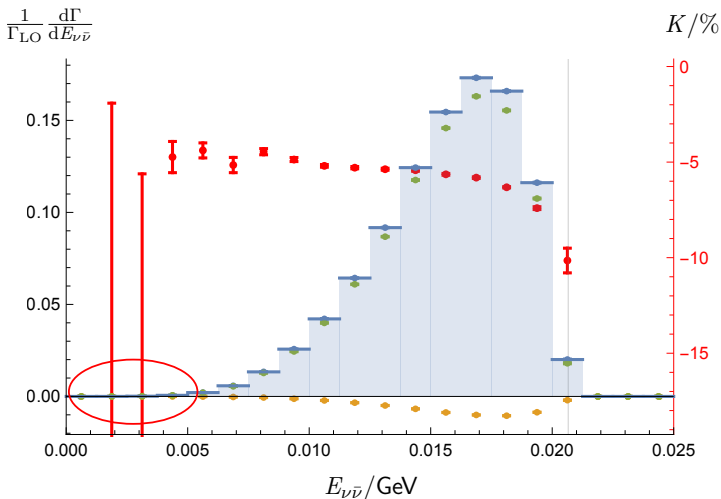


Fig.: Neutrino spectrum: Experimental resolution ≈ 2 MeV \Rightarrow low energy neutrino are important (below 5 MeV: 7.2×10^{-4})

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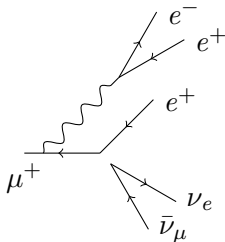
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Outlook

NLO branching ratios for $\text{Mu}3e$ @ PSI

- $4_{\text{Born}} + 40_{1\text{-loop}} + 20_{\text{real}}$ diagrams up to pentagons
- A lot but not *that* many
- Use same approach (GoSam, FKS, VEGAS)
- Phase space more important than ever
- $\text{Mu}3e$ cuts $E_e > 10 \text{ MeV}$



	LO	NLO only	K -factor
$\mathcal{B}(\text{no cuts})$	3.605×10^{-5}	0.007×10^{-5}	-0.19%
$\mathcal{B}(E > 10 \text{ MeV})$	2.309×10^{-6}	-0.041×10^{-6}	-1.78%

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Conclusion

- New fully differential NLO predictions for the radiative decay for MEG
- New NLO predictions for the rare decay for $\text{Mu}3e$

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- New NLO predictions for the rare decay for $\text{Mu}3e$

Work to be done

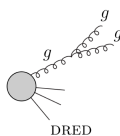
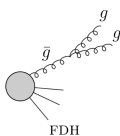
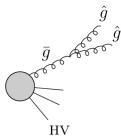
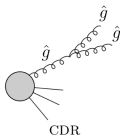
- Predict / Resum large logs $\log \frac{m}{M} \log \frac{\omega_0}{M}$ @ NLO and possibly NNLO
- ⇒ Solve 3.5σ discrepancy
- Produce distributions for the rare decay @ NLO

Regularization scheme dependence of two-loop amplitudes

└ Schemes

- 1 Variants of dimensional regularisation:
 - CDR ("conventional dimensional regularization")
 - HV ("t Hooft Veltman")
- 2 Variants of dimensional reduction:
 - DRED ("original/old dimensional reduction")
 - FDH ("four-dimensional helicity scheme")

	CDR	HV	FDH	DRED
internal gluon	$\hat{g}^{\mu\nu}$	$\hat{g}^{\mu\nu}$	$g^{\mu\nu}$	$g^{\mu\nu}$
external gluon	$\hat{g}^{\mu\nu}$	$\bar{g}^{\mu\nu}$	$\bar{g}^{\mu\nu}$	$g^{\mu\nu}$



$$g = Q4D, \quad \hat{g} = QD - \text{dim.}, \quad \bar{g} = 4 - \text{dim.}$$