ITP lunch seminar

Introduction

At fixed order LO NLO NNLO

$$\sigma = \int_n d\phi_n \left| A_n^{(0)} + A_n^{(1)} + A_n^{(2)} + \dots \right|^2 S(p_i)$$

$$+ \int_{n+1} d\phi_{n+1} \left| A_{n+1}^{(0)} + A_{n+1}^{(1)} + \dots \right|^2 S(p_i)$$

$$+ \int_{n+2} d\phi_{n+2} \left| A_{n+2}^{(0)} + \dots \right|^2 S(p_i)$$

need to use numerical integration for phase space as measurement function S too complicated / not known

real matrix element

$$|A_{n+1}^{(0)}|^2 \sim \Big|$$  $$+ \dots \Big|^2 \sim \frac{1}{E_\gamma^2} \frac{1}{1-\beta\cos\theta}$$

$$\xrightarrow{E_\gamma \to 0} \infty$$

for massive fermions $\cos\theta \to 1$ is fine as $\beta < 1$

How to do that in d-dim?

Subtraction & slicing

Idea 1 slicing

$$\int d\sigma_{n+1} = \int_0^{\omega_c} d\sigma_{n+1} + \int_{\omega_c} d\sigma_{n+1}$$

approximate & analytic

exact & numeric

$$= \frac{1}{\varepsilon} + \log \omega_c - \log \omega_c + \text{finite}$$

large cancellation as $\frac{2\omega_c}{\sqrt{s}} \approx 10^{-6}$

⇝ end up wasting most of your MC points on something that cancels anyway

Idea 2: subtraction

$$\int d\sigma_{n+1} = \int dCT + \int (d\sigma_{n+1} - dCT)$$

analytic

locally finite & well-behaved

(can be matched with slicing)

⇝ numeric

main advantage of subtraction:

+ local cancellation rather than global

(2-6x faster [2003.01014]
10x faster [Dittmaier])

− generally more complicated

First scheme @ NLO FKS '95, later CS '96

NNLO Antenna '07
STRIPPER '10
Projection-to-Born '15
residue-improved sec. decomp '17
Turin scheme '18

<u>side note</u>

$\int \frac{d^d k}{(2\pi)^d}$  $+ \int \frac{d^{d-1} k}{(2\pi)^{d-1} 2k^0} \, d\,CT = \text{finite}$

use loop-tree?

For QED $d\,CT$ & $\int d\,CT$ is known to all orders (FKS^e)

⇝ can calculate fixed-order cross-section & histograms to $N^e LO$

Event generation

experimentalists need generators, i.e. events $\{p_1, p_2, p_3, \ldots, p_n\}$, w, for detector sim.

trivial in principle, just dump all MC points to file (garden hose approach)

Problem: weights w not uniform, many orders of magnitude & negative

$$\int d\sigma_{n+1} = \underbrace{\int dCT}_{\approx 0} + \underbrace{\int (d\sigma_{n+1} - dCT)}_{\text{whatever}}$$

slicing actually has an advantage here...

not a problem for $d\sigma/dx$ b/c $\int$ is fast $\mathcal{O}(\mu s)$

detector sim $\int$ $\mathcal{O}$(min - hours)

$\rightsquigarrow$ minimise # of events

estimate: say we have N events $w = \pm c$
$r \cdot N$ of which are negative.

variance

$$\sim \frac{\sum_i w_i^2}{\left(\sum_i w_i\right)^2} = \frac{c^2 N}{[N(1-r)c - Nrc]^2} = \frac{1}{N} \frac{1}{(1-2r)^2}$$

need $1/(1-2r)^2$ more points...

LHC example

$pp \to$	$\gamma\gamma$ + jets	5x
	Z + 1 jet	2500x
	Z + 2 jet	6000x
	Z + 3 jet	9000x
	W + 5 jet	14000x

need to reduce r !

Cellular resampling

Andersen, Maier '21

Andersen, Maier, Maître '23

github.com/a-maier/cres

Two observations:

- cross sections are positive

  $\forall$ Region $\int_R d\sigma > 0$

- experiments have finite resolution

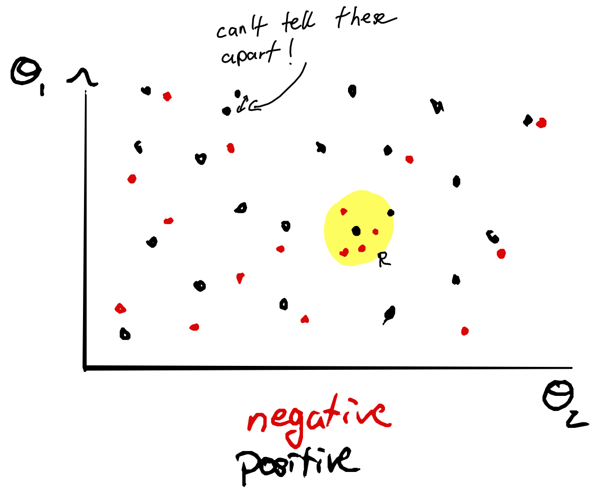
  $\rightsquigarrow$ KLN

  $\forall$ event e_1 $\exists$ e_2 s.t. e_1 & e_2 are indistinguishable

define a distance in event space

eg.

$$d(e^{(1)}, e^{(2)})^2 = [\theta_1^{(1)} - \theta_1^{(2)}]^2 \; [\theta_2^{(1)} - \theta_2^{(2)}]^2$$



algorithm

1 pick an event with $w<0$
(the seed)

2. add nearest event to cell C
3. if $\sum_{i \in C} w_i < 0$ goto 2
if size of C > resolution, goto 5

4. $$w_i \to \frac{\sum_{j\in C} w_j}{\sum_{j \in C} |w_j|} |w_i| \quad \forall i \in C$$

5 add more events in cell
& try again

Step 3: cancel things locally rather than globally (subtr. vs slicing)

Step 5: only possible if part of generator (cf. McMule)
guaranteed to work if enough points regardless of resolution

Summary

cancel things early rather than late